DOCUMENT RESUME

ED 199 089 SE 034 439

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| AUTHOR | Biemiller, Andrew |
| TITLE | A Neo-Piagettian Approach to Development During the Formal Operational Period. |
| PUB DATE | Jan 81 |
| NOTE | 27p.: Paper presented at the Annual Meeting of the International Interdisciplinary Conference on Piagetian Theory and Its Implications for the Helping Professions (11th, Los Angeles, CA, January 30-31, 1981). Contains occasional marginal legibility. |
| EDRS PRICE | MF01/PC02 Plus Postage. |
| DESCRIPTORS | *Cognitive Development; Cognitive Processes; *Educational Research; Elementary Secondary Education; Intelligence; *Learning Theories; *Mathematics Education; Memory; *Problem Solving; Psychological Testing |
| IDENTIFIERS | *Mathematics Education Research; Piagetian Stages; *Piagetian Theory |

ABSTRACT

Cognitive development during each of the major stages identified by Piaget is characterized by abilities to solve progressively more complex tasks (e.g., changes in the object concept during the sensori-motor stage, and in conservation of amount, weight, and volume during the concrete operational stage). Several theorists have suggested that these changes could be explained by increases in working memory or information processing capacity. Case (1978) suggests that development within each of the major Piagetian stages reflects the growth of working memory for the class of cognitive operations which become available to the child in the stage. The hypothesis that working memory for "formal" cognitive operations increases from one at ages 10 to 12 to four at ages 17 or 18 is examined. Evidence from prior studies of formal operational thought is reviewed. Developmental data using several measures of formal operational memory including tasks involving anticipation of elements in a classification matrix, solution of algebraic problems, and ratio relationships is presented. In general, results are consistent with the hypothesis that there is development of a formal operational working memory. Implications for instruction and other aspects of adolescent development are discussed. (Author)

A Neo-Piagettian Approach to Development

During the Formal Operational Period

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Paper delivered at the Eleventh Annual International Interdisciplinary Conference on Piagetian Theory and Its Implications for the Helping Professions. Los Angeles, Cal.: University of Southern California January 30, 1981.



Running head: Formal Operational Development

# A Neo-Piagettian Approach to Development During the Formal Operational Period

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## Abstract

Cognitive development during each of the major stages identified by Piaget is characterized by abilities to solve progressively more complex tasks (e.g., changes in the object concept during the sensori-motor stage, and in conservation of amount, weight, and volume during the concrete operational stage). Several theorists have suggested that these changes could be explained by increases in working memory or information processing capacity. Case (1978) suggests that development within each of the major Piagettian stages reflects the growth of working memory for the class of cognitive operations which become available to the child in the stage. In this paper, the hypothesis that working memory for "formal" cognitive operations increases from one at ages 10 to 12 to four at ages 17 or 18 will be examined. Evidence from prior studies of formal operational thought is reviewed. Developmental data using several measures of formal operational memory including tasks involving anticipation of elements in a classification matrix, solution of algebraic problems, and ratio relationships is presented. In general, results are consistent with the hypothesis that there is development of a formal operational working memory. Implications for instruction and other aspects of adolescent development are discussed.

A Neo-Piagettian Approach to Development
During the Formal Operational Period

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This exploratory study concerns an initial test of the hypothesis that a "working memory" for "formal" cognitive operations develops between ages 10 to 18. Case (1978) has suggested that developmental changes during the period of formal operational development will parallel the pattern of development he and others have described in younger children. Case and his students have demonstrated that "horizontal decalages" or the changes in abilities within the sensori-motor, pre-operational and concrete operational stages can be described by a growth in working memory for the operations characterizing each stage from approximately 1 "chunk" to approximately 4 "chunks". (Case, 1978; Case and Khanna, in press; Case, 1977; Case and Kurland, in press.) Thus it seems plausible that a comparable pattern of development would be observed during formal operational development.

Examination of the original studies of Piaget and Inhelder supports this hypothesis. Inhelder and Piaget observe that adolescent or "formal" thought is characterized by the generation of "possibilities" or "theories". (Inhelder and Piaget, 1958, p. 339). If Case's hypothesis is correct, we would expect that the maximum number of dimensions or abstract concepts that could be coordinated in constructing a "theory" would grow from one at approximately age 10 to four at approximately age 17 or 18. (This number can be expanded by the use of written symbols.) The "substages" described by Inhelder and Piaget appear to illustrate this quantitative growth in formal operational capacity. For example, in the much cited "separation of variables" problem, in which a child must anticipate what combination of characteristics will yield the most flexible rod, they report that early formal thinking (around age 12) is characterized by the inability to coordinate all (five) relevant factors, despite some understanding of the existence of dimensions of variability in rods. They describe an older child (age 16) as successfully coordinating four characteristics. (Inhelder and Piaget, 1958, pp. 56-60.) In another procedure, "the hydraulic press," three variables need to be coordinated - weight on the press, height of fluid in a connected column, and desnity of fluid. Interestingly, they indicate that this formal task is mastered rather earlier than the separation of variables problem. (Inhelder and Piaget, 1958, pp. 148-160.) In another, the balance beam, with two variables (length and weight) solutions are reported at a still earlier age. (Inhelder and Piaget, 1958, pp. 173-174.)

While ex post facto interpretations of data are interesting, empirical demonstrations of the growth of a formal operational working memory would be more convincing. Measures of working memory differ from other measures used to delineate or describe cognitive levels (e.g. conservation, separation of variables) in that a particular operation will be repeated an increasing number of times and must be held in "storage" while being repeated. (For example, in backward digit span - a measure of concrete operational working memory - we systematically increase the number of items which must be reversed.) In the present stu , formal operational working memory was assessed using three procedures: anticipation of elements of a classification scheme (number of defining criteria that had to be coordinated varied systematically); combination of algebraic operations (with the number of operations combined varied systematically); and solution of Noelting's (1975) juice problems with involve increasingly complex ratio computations for solution. Children at age levels hypothesized to reflect formal operational working memory capacities of 1 to 4 were given the tests. On the basis of Inhelder and Piaget's (1958) data, I have expected children at age 10 to 11 years to demonstrate a capacity of 1 unit of formal operational working memory (M). Children of age 13 were expected to demonstrate a formal operational working memory of M=2, while 15 year olds were expected to demonstrate 3 units and 18 year olds 4 units.

A strong confirmation of the hypothesis that a working memory for formal operations develops during adolescence would be a finding that children can solve correctly all problems within their working memory capacity, while failing or operating at chance levels on problems exceeding their capacities. This assumes, unrealistically, that my analysis of working memory requirements is correct, and that all children of a given age have the same memory. A weaker hypothesis would simply be that results will approximate the pattern described above - i.e. that hypothetical low working memory children will have trouble with tasks involving storage of several operations even though they can handle the same operations when memory requirements are less.

## Methods

All children studied attended the laboratory elementary and high schools of the University of Toronto's Faculty of Education. Level 1 children were 19 10 year olds (grade 4) and 16 11 year olds (grade 5). Average IQ for this population is around 120. Level 2 children were 22 13 year olds (grade 7). Level 3 children were 31 15 year olds (grade 9). Level 4 children were 21 18 year olds (grade 12). Average IQ for the level 2, 3, and 4 children is over 130. (No specific IQ data is available, but children at University of Toronto Schools (laboratory high school) are selected on the basis of academic proficiency.)

### Measures

All tests were given in pencil-and-paper format to classroom groups.

The classification anticipation task involved drawing forms in open cells of matrices on defining dimensions which were identified for the child and illustrated in the matrix. For example, a 3X3 matrix might differ in form (triangle, square, circle) and line (solid, broken, dotted). The child had to draw forms in 3 cells of the matrix. This represented coordination of two variables. Three, four and five variable matrices were also provided. In each case, the child had to draw 3 forms. Items were scored for the number of variables correctly included in each drawing. A total of 12 items were given. A sample 2 attribute problem is illustrated in Figure 1.

Figure 1 about here

Analysis of formal operational working memory requirements for this task is quite straightforward. The number of dimensions which must be coordinated to produce a correct figure was considered to reflect the working memory demand of the test.

The <u>algebraic coordination task</u> involved application of a symbol (X) which stood for a formal operation (e.g. (a times b - 1). Thus 7 (X) 8 means (7 × 8) - 1 or 55. Problems which required combinations of increasing numbers of this operation were considered to have increasing formal operational memory loads. Items used in this test are shown in Figure 2.

Figure 2 about here

Hypothetical analysis of working memory requirements for each item are as follows:

Item 1. One formal operation carried out and stored before recording. M = 1.

Item 2. Two formal operations to be carried out. One must be stored while the other is computed before comparison can be made. M = 2.

Item 3. Two formal operations to be carried out. Results of 1 must be stored (either the numerical result of the left side of the equation or 8Y - 1 for the right side) while the second operation is carried out. The equation is then solved for Y. M = 2.

Item 4. An additional operation (addition) is applied on each side of the equation. Thus two operations must be carried out on each side, while the result of the other side is stored. M = 3.

Item 5. Same analysis as item 4. M = 3.

Item 6. Same analysis as item 4 except that the final results must be solved for Y rather than compared. M = 3.

Item 7. The standard formal operation (X) is embedded within itself, (e.g. 3 X (5 X 2). We expected this to require an M of 3 for each side or 4 counting the stored result of one side while the other was processed. M = 4.

Item 8. Item 8 involves solving for an unknown value on each side of the equation. This involves storing equations with unknowns for each side (7y - 1, 9y - 9) and then solving for the unknowns (9y - 7y) and (-1 + 9). M = 4.

<u>The Noelting juice comparison problem</u>. This task involved determining which of two combinations of juice and water would have the greater concentration of juice or if the two were equal. All tasks involve computing and comparing ratios, (e.g. 2 parts juice and 1 part water vs. 3 parts juice and 4 parts water). Increased memory requirements were anticipated when additional ratio computations would be needed in order to make comparisons (e.g. 4 parts juice and 8 parts water vs. 2 parts juice and 4 parts water - one must be reduced to the common denominator for comparison). A sample item is illustrated in figure 3.

Figure 3 about here

The hypothesized analysis of working memory requirements for the Noelting Juice Tasks (based on Case, ref. note 1) is as follows:

Substage I: A simple determination of "more water", "more juice" or "equal" is made for sets A and B, (e.g. dimensional operation is performed for each set). Comparing these (e.g. operating on the results of two operations) involves 1 unit of formal operational working memory.[1]

Item 1. $\frac{A}{JJW} : \frac{B}{JJJWWWW}$ (more J vs. more water).

Substage II: Children can compare ratios of juice to water which can both be reduced to either unit numerates or unit denominators. These require storing a formal operation for each of set A and B.

Item 3. $\frac{A}{JWW} : \frac{B}{JJWW}$

Substage III (A). Children can compare ratios when one side cannot be reduced to unit form, (dealing with the deviation from unit ratio is considered to require an extra unit of working memory).

Item 6. $\frac{A}{JWW} : \frac{B}{JJJWWWWW}$ $(\frac{1}{2} : \frac{3}{5})$

Substage III (B) Children can compare ratios when neither side will reduce to unit form, but one side can be reduced or increased to the other's numerator or denominator.[1]

Item 13. $\frac{A}{JJWWW} : \frac{B}{JJJJJJWWWWWWWWW}$ $(\frac{2}{5} : \frac{6}{10} = \frac{4}{10} : \frac{6}{10})$.

Substage IV Children can compare ratios when there is no common numerator or denominator. This involves first determining each ratio and then changing each to a common denominator or numerator.

Item 16. $\frac{A}{JJJJJWW} : \frac{B}{JJJJJJJWWW}$ $(\frac{5}{2} : \frac{7}{3} = \frac{5}{6} : \frac{14}{6})$

## Results

Classification Anticipation Task. Table 1 shows percentages of correct responses by number of attributes to be coordinated (M load) and age. While not yielding perfect results (e.g. 100% performance on task at or below hypothesized memory load and 0% above) results clearly indicate a relationship between age and memory load.

Table 1 about here

Responses to the classification anticipation task can also be scored for the number of attributes correctly coordinated. This provides a further test of working memory spans on the test. Table 2 indicates that the maximum number of attributes correctly coordinated was limited to about 1.7 for the level 1 children, 2.9 for the level 2 children, 3.0 for the level 3 children, and 3.9 for the level 4 children. The two older groups approximated expected values. The level 2 group approximated expected values except on the 5 dimensional problem. The level 1 group exceeded the expected value. (There was no difference between grade 4 and 5 children.)

Table 2 about here

The Algebraic Combination Tasks. Two types of problems were given, (see figure 1). One type involved determination of inequalities ( $\gtreqless$ ) and were in effect multiple choice problems. The other involved solving equations for an unknown value. Table 3 presents results for these types of problems separately.

Table 3 about here

Results for inequality problems are not consistent with hypothesized outcomes. Nearly all children solved these problems at all levels except for the youngest children. Results for problems involving unknowns are more consistent with hypothesized working memory demands of the problems and age levels, although on the whole, the children did somewhat better than expected.

The Noelting Juice Task. Table 4 presents results for the Noelting Juice Task. This is a multiple-choice task with a chance level of 33% correct. Overall, results indicate that the level 1 tasks are solvable by most children as expected, and the level 4 tasks discriminate to some degree between children of different ages and are more difficult than the level 2 and 3 tasks. However, there is no discrimination in difficulty between average performance on level 2 and 3 tasks. Furthermore, there was a wide range of performance on individual items in these groups suggesting that the analysis of working memory requirements for these items needs to be revised. This matter will be taken up in the discussion.

## Discussion

Two of the three working memory tasks used yield results generally consistent with the hypothesis that working memory for formal operations increases with age during adolescence, although relatively high levels of performance indicate that the ages at which working memory for formal operations increases may be lower than hypothesized in this study. It is also clear that there are considerable individual differences in the development of formal operational working memory. (Note, for example, that 10 percent of 10 to 13 year old children solved classification anticipation problems involving coordination of four attributes, while 41% of 13 year olds solved a computational problem involving four operations.)

The results on the Noelting Juice Task indicated the need for a revised analysis of memory requirements. Examination of correct responses on all items suggests the following analysis of working memory requirements for ratio problems.

Substage 1. (Unchanged) A simple determination of "more juice", "equal", or "more water" is made for each set. The two are compared. (M = 1).

Substage 2. The ratio of juice to water is quantified for each set (e.g. $\frac{2J}{4W} \gtreqless \frac{3J}{8W}$ ). If the numerator or denominator of one ratio can be equated to the other (e.g. $\frac{2}{4} = \frac{4}{8} \gtreqless \frac{3}{3}$), this operation adds 1 unit of working memory. (M = 2).

Substage 3. If one of the two ratios can be reduced to 1 in either the numerator or denominator, it can then be equated to the other ratio, (e.g. $\frac{6J}{3W} \gtreqless \frac{5J}{2W} = \frac{2J}{1J} \gtreqless \frac{5J}{2W} = \frac{4J}{2W} \gtreqless \frac{5J}{2W}$ ).

This involves two operations adding another unit of working memory. (M = 3).

Substage 4. If there are no common denominators, a least common denominator must be calculated and both ratios changed to that value. (e.g. $\frac{5J}{2W} \gtreqless \frac{7J}{3W} = \frac{15J}{6W} \gtreqless \frac{14J}{6W}$). This adds another operation bringing M to 4.

Percentages of correct responses to items classified of this basis are given in table 5. This reanalysis yields expected results for the level 1 children (age 10 and 11). Somewhat high percentages of correct responses are still obtained by level 2 and 3 children. This may be due to the small values involved in one M=3 problem ($\frac{3J}{6W} \gtreqless \frac{2J}{3W}$) and one level 4 problem ($\frac{2J}{3W} \gtreqless \frac{3J}{4W}$) which may have been solved by simple knowledge of the relative magnitudes of the fractions (e.g. "three-fourths is more than two-thirds").

## Implications

The present data provide some support for the hypothesis that growth in formal operational working memory occurs during adolescence. If this hypothesis is born out in subsequent research, the significance of working memory requirements in applied formal operational tasks should be considered. For example, the development of advanced writing and mathematics skills, and many types of problem solving expected of adolescents probably require adequate levels of formal operational working memory. Similarly, the development of "identity" (Erikson, 1968, and others view identity processes as essentially the construction of a theory of oneself) would be influenced by working memory development. In short, attention to the implications of working memory in adolescence may have many of the same practical advantages as Case has outlined for working memory with the concrete-operational child. (Case, 1975).

The hypothesis that formal operational working memory is a developmental characteristic also has implications for the general development of formal operational capacity. Studies of the ages at which formal operational problems can be learned (e.g. Kuhn, Ho, and Adams, 1979; Danner and Day, 1977; and Stone and Day, 1978) indicate the existence of a growing age-related potential to acquire formal operational skills. Formal operational working memory may prove to be related to this potential just as concrete operational working memory has been shown to be related to the potential to acquire conservation skills. (Case, 1977).

Footnotes

I am indebted to my friend and colleague, Professor Robbie Case, for advice, support, and use of his algebraic combination task on this project. I am also indebted to Ms Debra Sandlos who administered the tests; and to the Headmaster, Donald Gutteridge and students of the University of Toronto Schools, and the Principal, Gery Mabin, and students of the Laboratory School of the Institute of Child Study.

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1. Analysis of this level differs from Case's (ref. note 1).

Reference Notes

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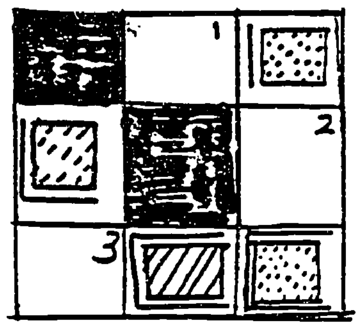
ANTICIPATING CLASSIFICATIONS

Instructions

This task requires you to find out what some members of a set of figures are. Look at practice set "A" below. There are four figures shown, all of which differ in two ways. First, they can have one, two, or three lines outside the figure; second inside the figure can be solid lines, dashed lines, or dotted lines. (In set "A", two squares are blacked out like a crossword puzzle).




Set A

In this problem, the inside lines are dashed in the first column, solid in the second column, and dotted in the third column. There is one outside line in the first row; two outside lines in the second row, and three outside lines in the third row. Using this information, try to draw the figures that should be in the spaces 1, 2, and 3. After you have drawn these, turn to page 2 and compare your answers. Feel free to write on the paper if that will help you solve any problem. If your answers differed from those given, read the explanation given below. Otherwise go on to page 3.

Figure 1. Instructions and example for the Classification Anticipation Task.

## Instructions

A new mathematical sign has been invented. The sign is $\otimes$, which means multiply and then subtract one. For example, $4 \otimes 2 = (4 \times 2) - 1 = 7$. Below are some questions which use this symbol. Please place your answer in the space provided

## Questions

1) $6 \otimes 5 =$ ____

2) $7 \otimes 5 \gtreqless 6 \otimes 8$, ____

3) $8 \otimes 2 =$ ____ $\otimes 8$

4) $2 + (7 \otimes 4) \gtreqless 3 + (4 \otimes 7)$, ____

5) $6 + (5 \times 4) \gtreqless 7 + (4 \otimes 5)$, ____

6) $(8 \otimes Y) - 8 = 6 + (6 \otimes 7)$, $Y =$ ____

7) $3 \otimes (5 \otimes 2) \gtreqless 3 \otimes (4 \otimes 2)$, ____

8) $(9 \otimes y) - 8 = y \otimes 7$, $y =$ ____

Figure 2. Instructions and items for the Algebraic Combination Task.

## INSTRUCTIONS

On each of the following pages are two sets of glasses which are filled with juice or with water . You must decide which set, when mixed together in a pitcher, would taste more strongly of juice.

The alternatives are that:

1) A will taste stronger

2) B will taste stronger

3) A and B will taste the same

1

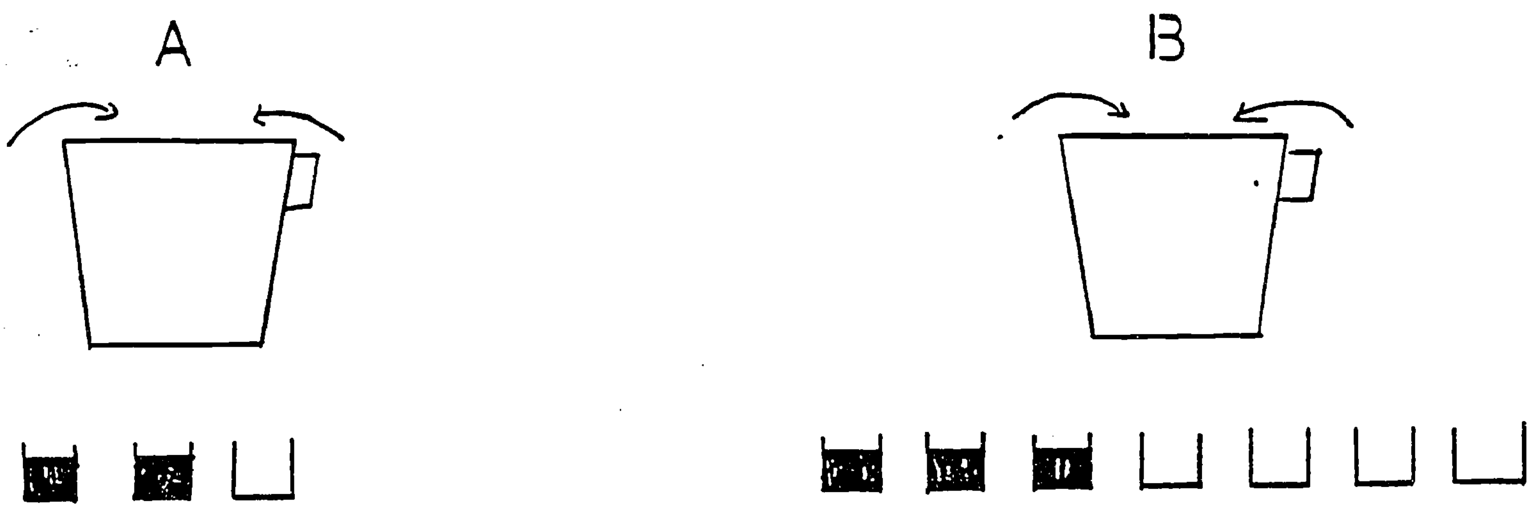



Figure 3. Instructions and example from the Noelting Juice Task.

Table 1. Percentage of Correct Responses by Age and Potential Number of Attributes to be Coordinated (Number of children in parentheses)

| | | Number of Attributes to be Coordinated | | | |
|---|---|---|---|---|---|
| Age | Hypothetical Working Memory | 2 | 3 | 4 | 5 |
| 10 or 11 | 1 | 57% (27) | 10% (21) | 10% (10) | 0% (10) |
| 13 | 2 | 88% (19) | 18% (19) | 10% (14) | 5% (13) |
| 15 | 3 | 94% (26) | 56% (28) | 34% (21) | 19% (18) |
| 18 | 4 | 96% (16) | 68% (16) | 74% (14) | 26% (14) |

Table 2. Mean Number of Attributes Coordinated by Age and Potential Number of Attributes to be Coordinated. (Number of children in parentheses)

| | | Number of Attributes to be Coordinated | | | |
|---|---|---|---|---|---|
| Age | Hypothetical Working Memory Capacity | 2 | 3 | 4 | 5 |
| 10-11 | 1 | 1.53(27) | 1.74(21) | 1.93(10) | 1.45(10) |
| 13 | 2 | 1.82(19) | 1.63(19) | 2.00(14) | 2.92(12) |
| 15 | 3 | 1.90(26) | 2.39(28) | 2.75(21) | 2.98(18) |
| 18 | 4 | 1.96(16) | 2.60(16) | 3.62(14) | 3.91(14) |

Table 3

| | Level | N (Item #) | Inequality Problems 2 (2) | 3 (4) | 3 (5) | 4 (7) | Computation Problems 1 (1) | 2 (3) | 3 (6) | 4 (8) |
|---|---|---|---|---|---|---|---|---|---|---|
| 10-11 | 1 | 35 | 77% | 84% | 52% | 56% | 100% | 67% | 33% | 9% |
| 13 | 2 | 22 | 86% | 91% | 82% | 100% | 100% | 100% | 59% | 41% |
| 15 | 3 | 31 | 90% | 90% | 94% | 97% | 100% | 100% | 81% | 71% |
| 18 | 4 | 21 | 95% | 91% | 81% | 100% | 95% | 100% | 91% | 95% |

Table 4. Percentages of Correct Responses on the Noelting Juice Task by Hypothesized Working Memory Demands and Age

| Age | Hypothesized Working Memory Capacity | N | Hypothesized Working Memory Demand 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|
| 10-11 | 1 | 35 | 83% | 48% | 47% | 31% |
| 13 | 2 | 22 | 93% | 78% | 78% | 64% |
| 15 | 3 | 31 | 97% | 98% | 89% | 74% |
| 18 | 4 | 21 | 100% | 99% | 97% | 93% |

Table 5. Percentage of Correct Responses on Revised Classification of Noelting Juice Tasks by Age

| Age | Hypothetical Working Memory Capacity | N | Working Memory Demand 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|
| 10-11 | 1 | 35 | 83% | 53% | 34% | 31% |
| 13 | 2 | 22 | 93% | 79% | 74% | 64% |
| 15 | 3 | 31 | 97% | 95% | 88% | 74% |
| 18 | 4 | 21 | 100% | 98% | 93% | 93% |